AN EFFICIENT DIGITAL FILTER DESIGN TOOL FOR APPROXIMATING AN FIR FILTER WITH A LOW-ORDER LINEAR-PHASE IIR FILTER

BACKGROUND OF THE INVENTION

1. Field of the Invention

The present invention relates to a method of designing digital filters, and more particularly, to a method of designing low-order linear-phase IIR filters for approximating an FIR filter.

2. Description of Related Art

Digital filters for addressing digital signal processing have been applied widely in commercial electronic products, such as compact disk players, television sets and the like. In order to deal with real-time signals, the designs should be considered low computational costs to output signals efficiently. Also, since the distortion-free transmission of the waveforms in the passband is very important in signal processing, the filters should contain a linear-phase characteristic, i.e., constant group delay.

There are two classes of digital filters. One is a finite-duration impulse response (FIR) filters and the other is an infinite-duration impulse response (IIR) filters. The main advantages of the FIR filters are exactly linear phase and guaranteed stable. However, when the specification being very rigorous, the resulting FIR filter is usually with higher orders, which may require more hardware components and lower the operational speed. Conversely, the IIR filters are useful for large-scale or high-speed designs but they do not have

exactly linear phase and can not guarantee to be stable.

One way to synthesize the IIR filters with linear phase in passband is to solve the rational approximation problem directly. These methods, for example, include Pade approximation, linear programming, nonlinear programming, multiple criterion optimization and eigenfilter approach.

Another way is called the indirect approach. It will be composed of three steps, as shown in Fig. 1. Step 1 receives and stores the design specifications of a digital filter.

These specifications are required in the frequency-domain in terms of the desired magnitude and phase response of the filter. Then, a linear-phase FIR filter, which meets design specifications, will be designed in step 2. The order and the coefficients of the FIR filter can be obtained using the conventional methods such as the frequency-sampling design technique, the window design technique and the optimal equiripple design technique. Finally, a lower-order IIR filter will be obtained using filter approximation techniques in step 3. It should be mentioned that the special attentions shall be paid on this indirect approach. The resulting IIR filter must be ensured to capture the linear-phase response of the original FIR filter in the passband.

In recent years, several linear-phase IIR filter design techniques have been emerged for this purpose. Generally speaking, two distinct methods have be proposed: (1) Grammian-Based Methods: including the balanced truncation method and impulse response grammian method and (2) Optimal Approximation Methods: including the least-square approximations and the

H_2 norm approximation. Although satisfactory results have been reported, computational complexity of these methods are still quite expensive.

SUMMARY OF THE INVENTION

The main objective of the present invention is to provide a water-preventing grommet for a pull chain switch, which efficiently keeps water out of the pull chain switch to avoid malfunction.

The secondary objective of the present invention is to provide a water-preventing grommet for a pull chain switch, which smoothes operations of the pull chain switch.

To achieve the objectives, the method of approximating an FIR filter with low-order linear-phase IIR filters by the rational Arnoldi algorithm with adaptive orders in accordance with the present invention contains the following steps:

(a) initialize the first vector of the Krylov sequence for each expansion point;

(b) in the jth iteration of the algorithm, choose an expansion frequency such that the frequency gives the greatest difference between the (j+1)st-order output moment of the original FIR filter $H(z)$ and that of the lower-order IIR filter $\hat{H}(z)$;

(c) after the chosen expansion point in jth iteration being determined, the single-point Arnoldi method applied at the expansion point to generate the new orthnormal vector;

(d) determine the new residual at each expansion point for the next

iteration; wherein the resulting orthogonal projection matrix is output after giving the total iteration number of the algorithm.

BRIEF DESCRIPTION OF THE DRAWINGS

Fig. 1 illustrates an indirect approach to design a low-order linear-phase IIR filter;

Fig. 2 illustrates the design flow of the low-order linear-phase IIR filters;

Fig. 3 illustrates the typical design specifications of a low-pass filter;

Fig. 4 shows the detail flow of the rational Arnoldi method with adaptive orders; and

Fig. 5A-7C show the bode plots of the magnitude, the error in magnitude, and the phase of the original FIR filters and the low-order IIR filters.

DETAILED DESCRIPTION OF THE PREFERRED EMBODIMENT

Figure 2 shows the design flow of a low-order linear-phase IIR filter in the present invention in which includes the steps of receiving and storing the design specifications in step 1, designing an FIR filter satisfying the design specifications and saving the order and coefficients in step 2, establishing the state space matrices $\{A,b,c\}$ in step 3, performing the rational Arnoldi method with adaptive orders and produce the orthogonal projection matrix V in step 4, and generating the corresponding low-order linear-phase IIR filter, which can approximate the original FIR filter and satisfy the design specifications in step 5.

Figure 3 illustrates typical design specifications of a low-pass filter in step 1, where the band $[0,\omega_p]$ (unit 34) is called the passband and δ_1 (unit 36) is the acceptable tolerance (or ripple) in the passband, the band $[\omega_s,\pi]$ (unit 38) is called the stopband and δ_2 (unit 40) is the corresponding tolerance (or ripple), and the band $[\omega_p,\omega_s]$ (unit 42) is called the transition band. R_p is the passband ripple in dB, where $R_p = -20\log_{10}[(1-\delta_1)/(1+\delta_1)]$. A_s is the stopband attenuation in dB, where $A_s = -20\log_{10}[\delta_2/(1+\delta_1)]$. Notably, either $\{\delta_1,\delta_2\}$ or $\{R_p,A_s\}$ is required to be stored in step 1.

Suppose that an FIR filter has been designed to satisfy the design specifications in step 2. Let $H(z)=\sum_{i=0}^{n} h_i z^{-i}$ be the causal FIR filter with length $n+1$. A state-space realization of $H(z)$ in step 3 can be described as

$$\begin{aligned} x(k+1) &= Ax(k) + bu(k) \\ y(k) &= c^T x(k) + h_0 u(k) \end{aligned}, \tag{1}$$

where

$$A = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix}, b = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ \vdots \\ 0 \end{bmatrix}, c = \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ \vdots \\ h_n \end{bmatrix}, \tag{2}$$

and $A \in R^{n\times n}, b \in R^n, c \in R^n$. The transfer function $H(z)$ can also be expressed as $H(z) = c^T X(z) + h_0 = c^T (zI_n - A)^{-1} b + h_0$. Our problem formulation is to find a lower-order IIR filter $\hat{H}(z)$, which satisfies the same specifications in step 1 as the original FIR filter $H(z)$ and maintains a linear-phase response in the passband.

The way in the invention is to find an optimal IIR filter by using orthogonal projection of the original FIR filter. By matching some characteristics of the original FIR filter, the resulting orthonormal matrix V can be generated in step 4. The lower-order IIR filter $\hat{H}(z)$ can be constructed using the orthonormal projection $x(k) = V\hat{x}(k)$. In such a situation, the parameters of the IIR filter can be defined by the following congruence transformation in step 5,

$$\hat{A} = V^T AV,\ \hat{b} = V^T b,\ \text{and}\ \hat{c} = V^T c. \tag{3}$$

It can be shown that the matrix $V^T AV$ is always stable as long as (1) and matrix A is stable, (2) $V^T V = I$. Thus, the stability of the lower-order IIR filter generated by Eq. (3) is guaranteed.

<u>Pade approximation and moment matching</u>

The basis theory of the method in the invention is the multi-point Pade approximation, or so called the multi-point moment matching, to obtain a low-order IIR filter. Expanding $X(z)$ in power series about various frequencies $\{z_1, z_2, \cdots, z_{\hat{i}}\}$, where each $z_i = e^{j\omega_i} \in C$ and $0 \le \omega_i \le \pi$, we have

$$X(z) = \sum_{j=0}^{\infty} X^{(j)}(z_i)(z - z_i)^j, \tag{4}$$

where

$$\begin{aligned} X^{(j)}(z_i) &= [-(z_i I_n - A)^{-1}]^j (z_i I_n - A)^{-1} b, \\ H^{(j)}(z_i) &= c^T X^{(j)}(z_i) \quad (j > 0), \\ H^{(0)}(z_i) &= c^T X^{(0)}(z_i) + h_0. \end{aligned} \tag{5}$$

$X^{(j)}(z_i)$ is called the jth-order system moment of $X(z)$; $H^{(j)}(z_i)$

represents the jth-order output moment of $H(z)$ at z_i. Notably, if $\hat{i}=1$, Eq. (4) is indeed the conventional Pade approximation. The objective is to find a q-order ($q < n$) IIR filter $\hat{H}(z)=\hat{c}^T(zI_q-\hat{A})^{-1}\hat{b}+h_0$ such that $H^{(j)}(z_i)=\hat{H}^{(j)}(z_i)$ for $j=0,1,\cdots,\hat{j}_i-1$ and $i=1,2,\cdots,\hat{i}$, where $q=\sum_{i=1}^{\hat{i}}\hat{j}_i$.

It shall be mentioned that moment calculations can be obtained analytically by exploring special characteristics of matrices A and b in eq. (2). For each z_i, $(z_iI_n-A)^{-1}b$ and $(z_iI_n-A)^{-1}$ can be derived analytically as the following formulas:

$$(z_iI_n-A)^{-1}=\begin{bmatrix} 1/z_i & 0 & \cdots & 0 & 0 \\ 1/z_i^2 & 1/z_i & \cdots & 0 & 0 \\ 1/z_i^3 & 1/z_i^2 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1/z_i^{n-1} & 1/z_i^{n-2} & \cdots & 1/z_i & 0 \\ 1/z_i^n & 1/z_i^{n-1} & \cdots & 1/z_i^2 & 1/z_i \end{bmatrix},$$

$$(z_iI_n-A)^{-1}b=\begin{bmatrix}1/z_i & 1/z_i^2 & \cdots & 1/z_i^n\end{bmatrix}^T.$$

Krylov subspace and the Arnoldi method

Explicitly computing moments usually yields numerically ill-conditioned problems. We adapt recent works about the Krylov space method to solve these problems. Given a square matrix $\Psi\in C^{n\times n}$ and a vector $\xi\in C^n$, the qth Krylov sequence

$$K_q(\Psi,\xi)\equiv\operatorname{span}(\xi,\Psi\xi,\Psi^2\xi,\cdots,\Psi^{q-1}\xi)$$

is a sequence of q column vectors and the corresponding column space is called the qth Krylov subspace. Set $\Psi=(z_iI_n-A)^{-1}$ and $\xi=(z_iI_n-A)^{-1}b$. It has been shown that the Krylov subspace $K_q(\Psi,\xi)$ is indeed spanned by the

system moments $X^{(j)}(z_i)$ for $j = 0,1,\cdots,q-1$. The Arnoldi method, a kind of Krylov subspace methods, is employed to generate an orthonormal matrix V_q that spans the same subspace as the Krylov subspace $K_q(\Psi,\xi)$. As a result, the guaranteed stable IIR filter can be constructed by substituting V_q into Eq. (3).

The Arnoldi method arises from the Hessenberg reduction $A = VHV^T$ for eigenvalue calculations. It has the advantage that it can be terminated part-way and leaving one with a partial reduction to a Hessenberg form. The process is exploited to form iterative algorithms. During the iteration process, an upper Hessenberg matrix $H_q \in C^{q\times q}$ is generated that satisfies the following relationship:

$$\Psi V_q = V_q H_q + h_{q+1,q} v_{q+1} e_q^T \text{ and } v_1 = \xi / \|\xi\|, \qquad (6)$$

where e_q is the qth unit vector in R^q. The vector v_{q+1} satisfies a (q+1)-term recurrence relation, involving itself and the preceding Krylov vectors. A new orthonormal vector v_{q+1} can be generated using the modified Gram-Schmidt orthogonalization technique.

The rational Arnoldi method

Generally speaking, the accuracy of the Pade approximation based methods is lost away from the expansion point more rapidly as the eigenvalues of the FIR filter approach the expansion frequency. A rational Arnoldi (RA) method, which uses multiple expansion points, was developed to overcome this difficulty. The straightforward way for multi-point moment matching applications is to apply the Krylov subspace algorithm at various

expansion frequencies. This is the so-called rational Krylov algorithm. Basically, this algorithm is a generalization of the shifted-and-inverted Arnoldi algorithm. To simplify the developments, the number of the matched moments of the lower-order IIR filter at each expansion point is assumed to be fixed. Formally, let $Z=\{z_1,z_2,\cdots,z_{\hat{i}}\}$ represent the set of predetermined expansion frequencies. Let $J=\{\hat{j}_1,\hat{j}_2,\cdots,\hat{j}_{\hat{i}}\}$ be the set of the number of the matched moments at each corresponding frequency. The rational Arnoldi method will generate a lower-order IIR filter $\hat{H}(z)$, which matches q-order ($q=\sum_{i=1}^{\hat{i}}\hat{j}_i$) moments of the FIR filter, $H(z)$, at the expansion points z_i, $i=1,2,\cdots,\hat{i}$.

Implementing the rational Arnoldi method is equivalent to implement the Arnoldi method $\hat{j}_i$ times at $\hat{i}$ expansion frequencies. That is, the first $\hat{j}_1$ iterations correspond to the expansion frequency z_1 and the next $\hat{j}_2$ iterations are associated with z_2, and so on. Each Arnoldi iteration generates $\hat{j}_i$ orthonormal vectors. Then, $V_q=\lfloor v_1 \quad v_2 \quad \cdots \quad v_q \rfloor$ is the desired orthonormal matrix generated from a union Krylov space at various expansion points, as stated by

$$\mathbf{K}_q=\mathrm{span}(X^{(0)}(z_1),\cdots,X^{(\hat{j}_1-1)}(z_1),\cdots\cdots,X^{(0)}(z_{\hat{i}}),\cdots,X^{(\hat{j}_{\hat{i}}-1)}(z_{\hat{i}})).$$

Once the orthonormal matrix V_q has been formed by applying the rational Arnoldi method and the lower-order IIR filter can be obtained using the congruence transformation.

<u>The rational Arnoldi method with adaptive orders</u>

Selecting a set of expansion points z_i for $i=1,2,\cdots,\hat{i}$ and the number

of matched moments $\hat{j}_i$ about each z_i is by no means trivial. For simplicity, the expansion points z_i for $i = 1,2,\cdots,\hat{\imath}$ are determined in advance using engineering heuristics or experimental measurements over a specified frequency range. This invention describes an intelligent scheme for choosing multiple expansion points in each of the iterations.

Suppose that $H^{(j)}(z_i) = \hat{H}^{(j)}(z_i)$ for $j = 0,1,\cdots,\hat{j}_i - 1$ and $i = 1,2,\cdots,\hat{\imath}$ after q iterations of the rational Arnoldi algorithm. However, the $\hat{j}_i$th-order output moments $H^{(\hat{j}_i)}(z_i) = \hat{H}^{(\hat{j}_i)}(z_i)$ can not be guaranteed. The concept that underlies the rational Arnodli method with adaptive orders is to select an expansion point $z_{i^*_{q+1}}$ in the (q+1)st iteration. Hence, the resulting (q+1)st-order IIR filter yields the greatest moment improvement $|H^{(\hat{j}_i)}(z_i) - \hat{H}^{(\hat{j}_i)}(z_i)|$ of the qth-order IIR filter as $z_i = z_{i^*_{q+1}}$. The moment errors can be directly obtained in the new iteration without explicitly calculating system moments.

Figure 4 displays the detail flow of the rational Arnoldi method with adaptive orders in step 4 in Fig. 2.

Step 1, in Fig. 4, initializes the first vector $k^{(0)}(z_i) = (z_i I_n - A)^{-1} b$ of the Krylov sequence for each expansion point z_i, where $i \in \{1,\cdots,\hat{\imath}\}$. Since the lower-order IIR filter and the orthonormal matrix are not yet determined, the residue $r^{(0)}(z_i)$ for each z_i is set to $k^{(0)}(z_i)$. The normalization coefficient about each z_i, $h_\pi(z_i)$, is initialized to be one. Step 2, in Fig. 4, begins the iterations and sets $j = 1$.

Step 3, in Fig. 4, chooses an expansion frequency z_i such that z_i

gives the greatest difference between the (j+1)st-order output moment of the original FIR filter $H(z)$ and that of the lower-order IIR filter $\hat{H}(z)$, that is,

$$\max_{z_i \in Z} |H^{(j+1)}(z_i) - \hat{H}^{(j+1)}(z_i)| = \max_{z_i \in Z} |h_\pi(z_i) c^T r^{(j-1)}(z_i)|.$$

$\hat{H}^{(j+1)}(z_i)$ is the (j+1)st-order output moment of the lower-order IIR filter $\hat{H}(z)$, which is yielded using the congruence transformation matrix V_{j-1} (j>1) and matches j-order output moments of $H(z)$ at z_i. The chosen expansion frequency in the jth iteration is called $z_{i_j^*}$.

After choosing the expansion point $z_{i_j^*}$ in the determined jth iteration, the single-point Arnoldi method is applied at the expansion point $z_{i_j^*}$ (unit 52), which contains steps 4 and 5, as shown in Fig. 4. Step 4, in Fig. 4, generates the new orthnormal vector v_j and the vector is incorporated into the orthnormal matrix V_{j-1}. The normalization coefficient $h_\pi(z_i) = \prod_j \left\| r^{(j-1)}(z_i) \right\|$ when z_i is selected in the jth iteration.

Step 5, in Fig. 4, determines the new residual $r^{(j)}(z_i)$ at each expansion point z_i. The calculation involves a projection with the new orthonormal matrix V_j. The next vector $k^{(j)}(z_{i_{q+1}^*})$ at the frequency $z_{i_{q+1}^*}$ must be updated to enable further matching of the output moment in the (j+1)st iteration. Since no improvement is obtained at the other unselected frequency z_i, the vector $k^{(j)}(z_i)$ at frequency z_i in the current iteration remains $k^{(j-1)}(z_i)$, which was obtained in the preceding iteration. Reset $j = j+1$ in step 6 and judge if $j \le q$ in step 7, as shown in Fig. 4. Finally, the resulting orthogonal projection matrix V_q is generated in step 8 in Fig. 4.

The resulting orthnormal matrix V_q should be real to ensure that real system matrices of the lower-order IIR filter are generated if the complex expansion frequencies are used. First, all column vectors in V_q are divided into the real part V_r and the imaginary part V_i. Second, a reduced QR factorization of $[V_r \quad V_i]$ is performed to yield a new orthogonal matrix V_q. The moment matching property of the resulting lower-order IIR filter by the new and real V_q is also preserved.

The details of the algorithm are outlined as follows. The vector Z includes $\hat{i}$ expansion points, q is the total number of iterations and V_q is the resulting orthonormal matrix.

Adaptive Rational Arnoldi (input: A,b,c,Z,q; output: V_q)

(1): /* Initialize */

1 for each $z_i \in Z$ **do**

2 $k^{(0)}(z_i) := (z_i I_n - A)^{-1} b$, $r^{(0)}(z_i) := k^{(0)}(z_i)$

3 $h_\pi(z_i) := 1$

4 end for

(2): /* Begin the Iterations */

5 for $j = 1,2,\cdots,q$ **do**

(2.1) /* Select the Expansion Frequency with the Maximum Output Moment Error*/

6 Choose $z_i \in Z$ as the i giving $\max_i |h_\pi(z_i) c^T r^{(j-1)}(z_i)|$

7 set $z_{i_j^*}$ be the expansion frequency in the jth iteration

(2.2) /*Generate the Orthonormal Vector at $z_{i_j^*}$ */.

$h_{j,j-1}(z_{i_j^*}) := \left\| r^{(j-1)}(z_{i_j^*}) \right\|$

$v_j = r^{(j-1)}(z_{i_j^*}) / h_{j,j-1}(z_{i_j^*})$

$h_\pi(z_{i_j^*}) := h_\pi(z_{i_j^*}) \cdot h_{j,j-1}(z_{i_j^*})$

(2.3) /* Update the Residue $r^{(j)}(z_i)$ for the Next Iteration */

for each $z_i \in Z$ **do**

  if ($z_i == z_{i_j^*}$) **then** $k^{(j)}(z_{i_j^*}) := -(z_i I_n - A)^{-1} v_j$

  else $k^{(j)}(z_i) := k^{(j-1)}(z_i)$

  end if

  $r^{(j)}(z_i) := k^{(j)}(z_i)$

    for $t = 1, 2, \cdots, j$ **do**

      $h_{t,j}(z_i) := v_t^H r^{(j)}(z_i)$

      $r^{(j)}(z_i) := r^{(j)}(z_i) - h_{t,j}(z_i) v_t$

    end for

end for

end for

$V_q = [v_1 \quad v_2 \quad \cdots \quad v_q]$

Some properties of the method of approximating an FIR filter by low-order IIR filters in the invention are summarized as follows.

(1) Exact expression of output moment errors: suppose that the output moments of the original FIR filter and those of the lower-order IIR filter are matched, that is, $H^{(j)}(z_i) = \hat{H}^{(j)}(z_i)$ for $j = 0, 1, \cdots, \hat{j}_i - 1$ and $i = 1, 2, \cdots, \hat{\imath}$. The system matrices of the lower-order IIR filter are generated by the

congruence transformation with the orthonormal matrix V_q using the algorithm, where $q = \sum_{i=1}^{\hat{i}} \hat{j}_i$. The magnitude error between the $\hat{j}_i$ th-order moments $H^{(\hat{j}_i)}(z_i)$ and $\hat{H}^{(\hat{j}_i)}(z_i)$ at each expansion point z_i can be expressed as follows:

$$|H^{(\hat{j}_i)}(z_i) - \hat{H}^{(\hat{j}_i)}(z_i)| = |h_\pi c^T r^{(\hat{j}_i - 1)}(z_i)|, \tag{10}$$

where $h_\pi(z_i) = \prod_j \left\| r^{(j-1)}(z_i) \right\|$.

(2) Moment matching can still be preserved.

(3) In the first iteration in the rational Arnodli algorithm with adaptive orders, step (2.2) is to choose $z_i \in Z$ such that $\max(|c^T (z_i I_n - A)^{-1} b|) = \max(|H(z_i)|)$. This is equivalent to find out the expansion frequency with the maximum magnitude in the output frequency response.

(4) Implementation issues of digital filters: the present invention also provides several heuristics of selecting expansion frequencies in advance for the proposed rational Arnoldi method. Generally speaking, the complex expansion points $\{z_1, z_2, \cdots, z_{\hat{i}}\}$ will be recommended, where each $z_i = e^{j\omega_i} \in C$ and $0 \le \omega_i \le \pi$. Then the frequency responses of the lower-order IIR filters at these points can be the same as those of the original FIR filter. Nevertheless, if real expansion points can be selected, the computational complexity of yielding approximate IIR filters can be further reduced. The following guidelines are provided:

(a) Low-pass/high-pass filters: the proposed method with the expansion point $\omega_1 = 0$ performs well over the low frequency range of

responses. For high-pass filter designs, the special structures of state-space matrices may be used to present the duality between low-pass and high-pass filters. Let $\overline{A} = -A$, $\overline{b} = b$, $\overline{c} = c$, and $\overline{h_0} = -h_0$,

$$\overline{H}(z) = \overline{c}^T (zI_n - \overline{A})^{-1}\overline{b} + \overline{h_0} = \sum_{i=0}^{n} (-1)^{i+1} h_i z^{-i} .$$

If $H(z)$ presents a high-pass filter, then $\overline{H}(z)$ will be a low-pass filter, and a vice versa. Likewise, the expansion point $\omega_1 = 0$ is chosen to perform the Arnoldi algorithm. If the corresponding orthonormal matrix $\overline{V}_q$ is obtained, then the high-pass IIR filter, which satisfies the same specifications as the original FIR filter, can be constructed as follows:

$$\hat{A} = \overline{V}_q{}^T A \overline{V}_q, \hat{b} = \overline{V}_q{}^T b, \text{and } \hat{c} = \overline{V}_q{}^T c .$$

(b) Band-pass/band-stop filters: experimental results indicate that the passband edge and stopband edge frequencies are appropriate candidate expansion points in meeting the specifications of the design. Other expansion points with uniform spacing are also recommend to be selected.

Design Examples

Three example filters are used to justify the proposed approach. Table 1 describes specifications of a low-pass filter, a high-pass filter, and a band-pass filter. The command *remez* in Matlab was used to design the FIR filters by the optimal equiripple technique. Table 2 lists the corresponding orders. Then, the approximate low-order IIR filters were generated by the proposed method and the balanced realization method (BAL). Table 2 shows the reduced orders and the expansion points used by the two methods.

Figures 5A-7C display the bode plots of the magnitude, the error in magnitude, and the phase of the original FIR filters and the low-order IIR filters. In figures 5A-7C, the responses of the original FIR filters are represented as thin solid lines. Those of the IIR filters, determined by the proposed method, are represented as thick solid lines -, and those determined by BAL method are plotted as thick dashed lines - -. The responses in the passband of the IIR filters are indistinguishable from those of the original FIR filters, independently of which the model reduction method is used. Simulation results imply that the performance of the proposed method is similar to that of the BAL method in the passband. The resulting lower-order IIR filters can actually preserve the linear-phase response of the original FIR filters. Nevertheless, in terms of computational efficiency, the Kylov subspace based methods generally outperform the BAL method.

Table 1 Filter design specifications

Specifications	Low-Pass	High-Pass	Band-Pass
Maximum passband attenuation (dB)	3	1	1
Maximum stopband attenuation (dB)	40	35	35
Lower passband edge (rad/s)	0	0.85π	0.3255π
Upper passband edge (rad/s)	0.285π	1π	0.3755π
Lower stopband edge (rad/s)	0.353π	0	0.6655π
Upper stopband edge (rad/s)	1π	0.78π	0.7155π

Table 2 Matched orders and expansion points for IIR filter designs

		Low-Pass	High-Pass	Band-Pass
FIR	Order	41	41	58
IIR	Order	24	17	36

	Expansion points	$\omega = 0$	$\omega = \{0.4\pi, 0.6\pi\}$

Conclusions

A rational Arnoldi method with adaptive orders for approximating FIR filters by low-order linear-phase IIR filters has been proposed. The developed method is very efficient in terms of computational complexity. Meanwhile, the lower-order IIR filter can truly reflect the dynamical features of the FIR filter and satisfies the original design specifications.

Although the invention has been explained in relation to its preferred embodiment, it is to be understood that many other possible modifications and variations can be made without departing from the spirit and scope of the invention as hereinafter claimed.